$$q = -\mathbf{K}\ grad\ h$$

Darcy's law

grad is a vector

$$grad = \frac{\partial}{\partial x}\delta 1 + \frac{\partial}{\partial y}\delta 2 + \frac{\partial}{\partial z}\delta 3$$

$$grad(h) = \frac{\partial h}{\partial x}\delta 1 + \frac{\partial h}{\partial y}\delta 2 + \frac{\partial h}{\partial z}\delta 3$$

***q* is a vector**

$$q = q_x\,\delta_1 + q_y\,\delta_2 + q_z\delta_3$$

Transient mass balance equation:

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} - W = -S_s\,\frac{\partial h}{\partial t}$$

Steady State Mass Balance Equation with W = 0

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} = 0$$

$$\boldsymbol{grad \bullet q = 0}$$

$$\nabla \bullet \boldsymbol{q} = 0$$ (del notation)

The dot product of *grad* and *q* is
div *q* = 0

$$\frac{\partial}{\partial x}(K_x \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(K_y \frac{\partial h}{\partial y}) + \frac{\partial}{\partial z}(K_z \frac{\partial h}{\partial z}) = S_s \frac{\partial h}{\partial t} - W$$

Anisotropic medium: K at a point varies with direction

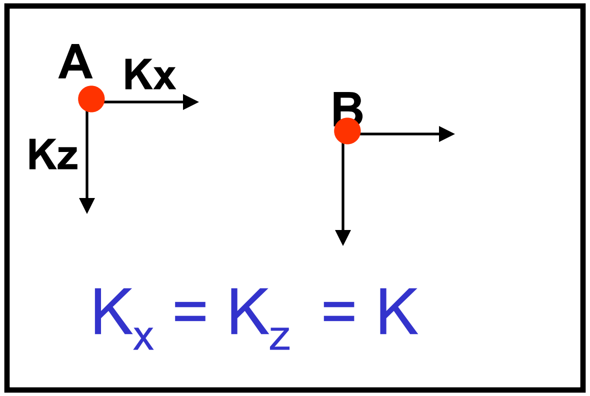
$K_x \neq K_y \neq K_z$

or

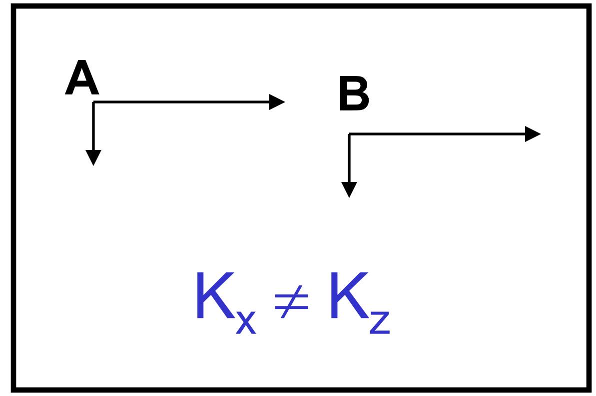
$K_x = K_y \neq K_z$

Heterogeneous medium: K varies in space

Characteristics of K in two dimensions

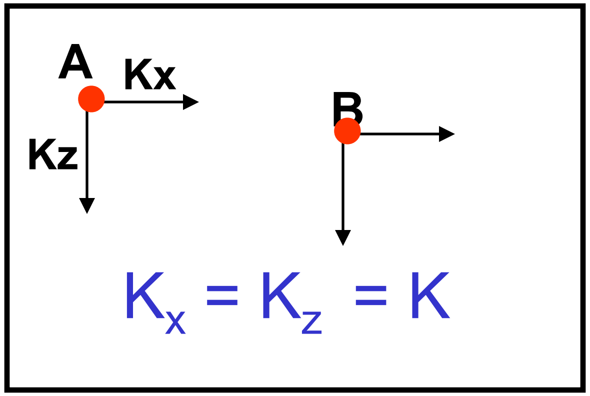


Homogeneous, isotropic

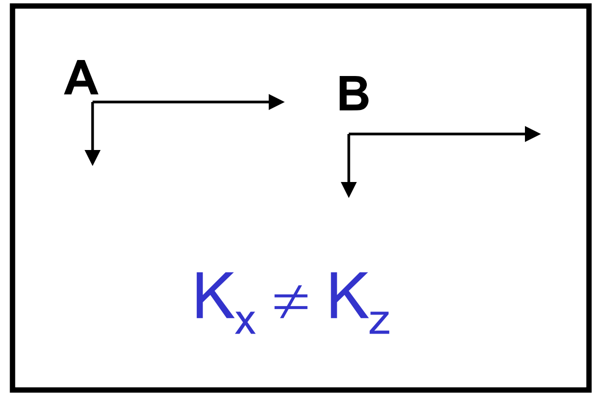


Homogeneous, anisotropic

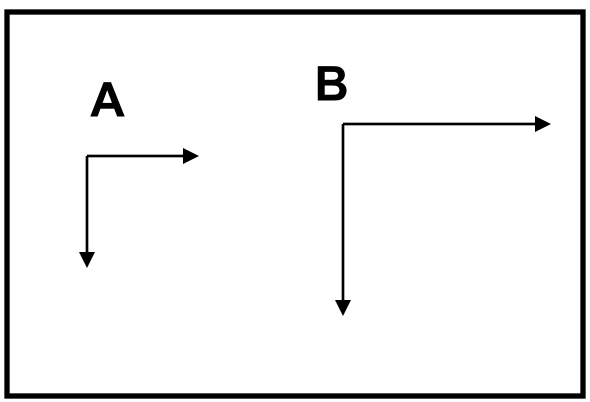
Characteristics of K in two dimensions



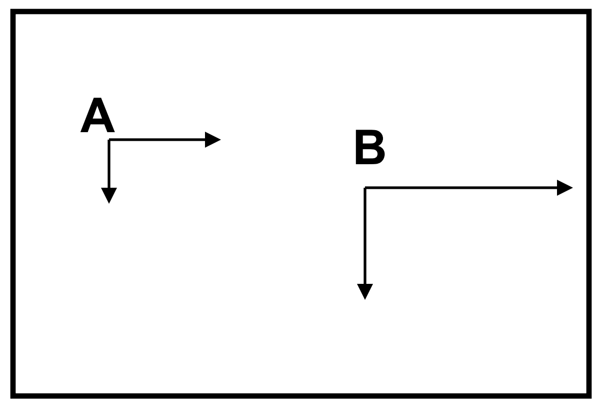
Homogeneous, isotropic



Homogeneous, anisotropic



Heterogeneous, isotropic



Heterogeneous, anisotropic

$$\frac{\partial}{\partial x}(K_x\frac{\partial h}{\partial x})+\frac{\partial}{\partial y}(K_y\frac{\partial h}{\partial y})+\frac{\partial}{\partial z}(K_z\frac{\partial h}{\partial z})=S_s\frac{\partial h}{\partial t}-W$$

homogeneous, isotropic porous medium: $K_x = K_y = K_z = K$

steady state conditions: $\frac{\partial h}{\partial t}=0$

W = 0

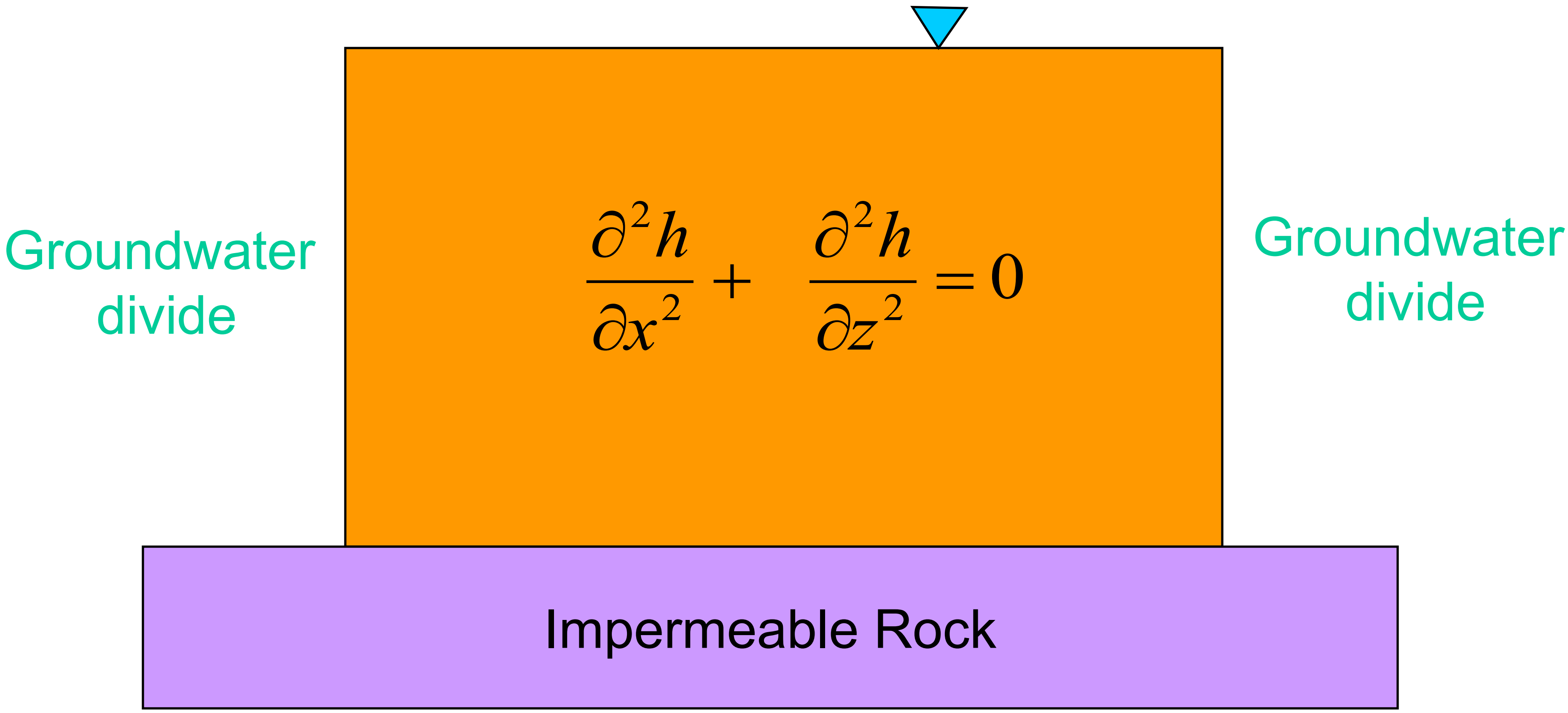
$$\frac{\partial}{\partial x}(\frac{\partial h}{\partial x})=\frac{\partial^2 h}{\partial x^2}$$

Laplace equation

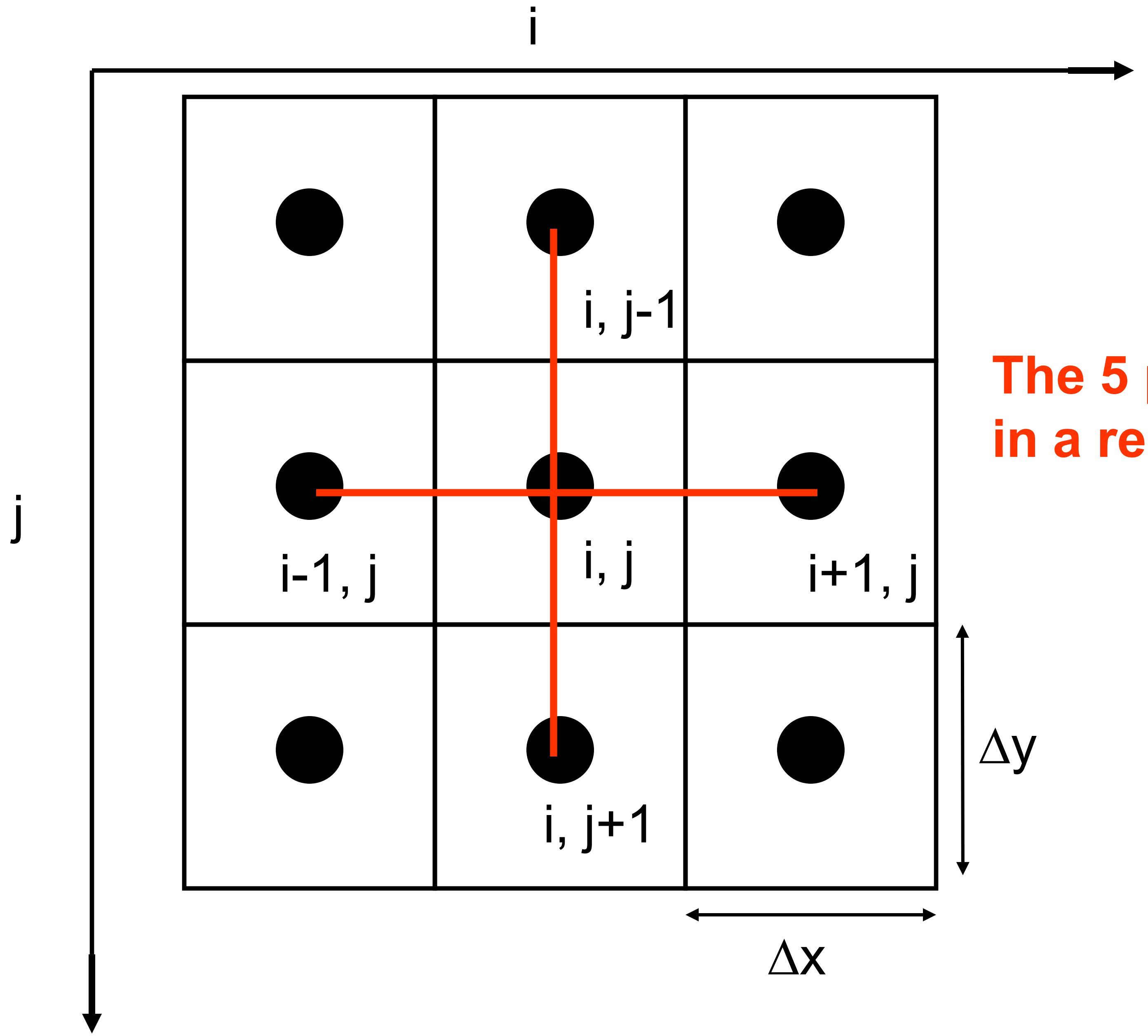
$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}=0$$

$$\nabla^2 h = 0$$

Toth Problem (2D)

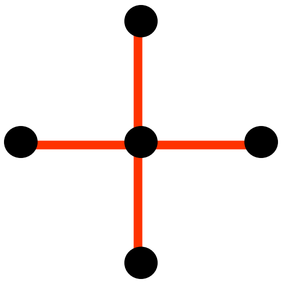


$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$



The 5 point star in a regular grid

$\Delta x = \Delta y$



$$h_{i+1,j} + h_{i-1,j} + h_{i,j+1} + h_{i,j-1} - 4h_{i,j} = 0$$

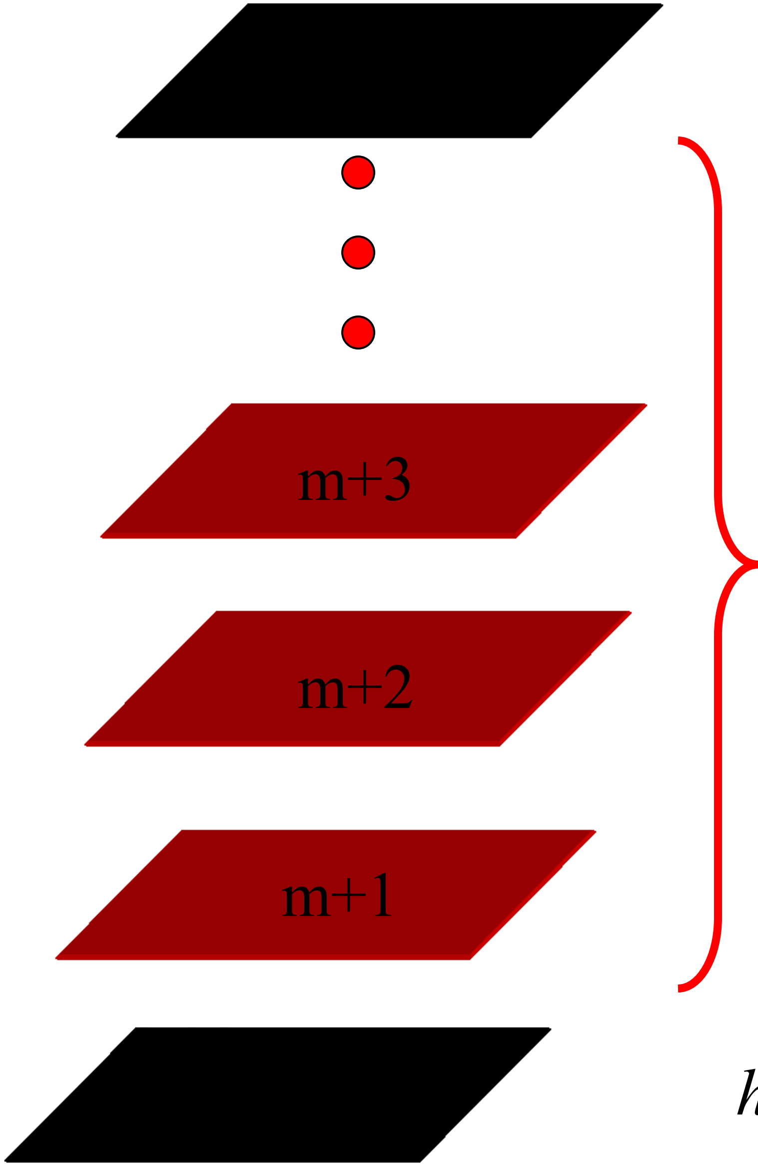
$$h_{i,j} = (h_{i+1,j} + h_{i-1,j} + h_{i,j+1} + h_{i,j-1})/4$$

Iteration indices

$$h_{i,j}^{m+1} = \frac{h_{i+1,j}^{m} + h_{i-1,j}^{m+1} + h_{i,j+1}^{m} + h_{i,j-1}^{m+1}}{4}$$

Solution by iteration

Gauss-Seidel Iteration

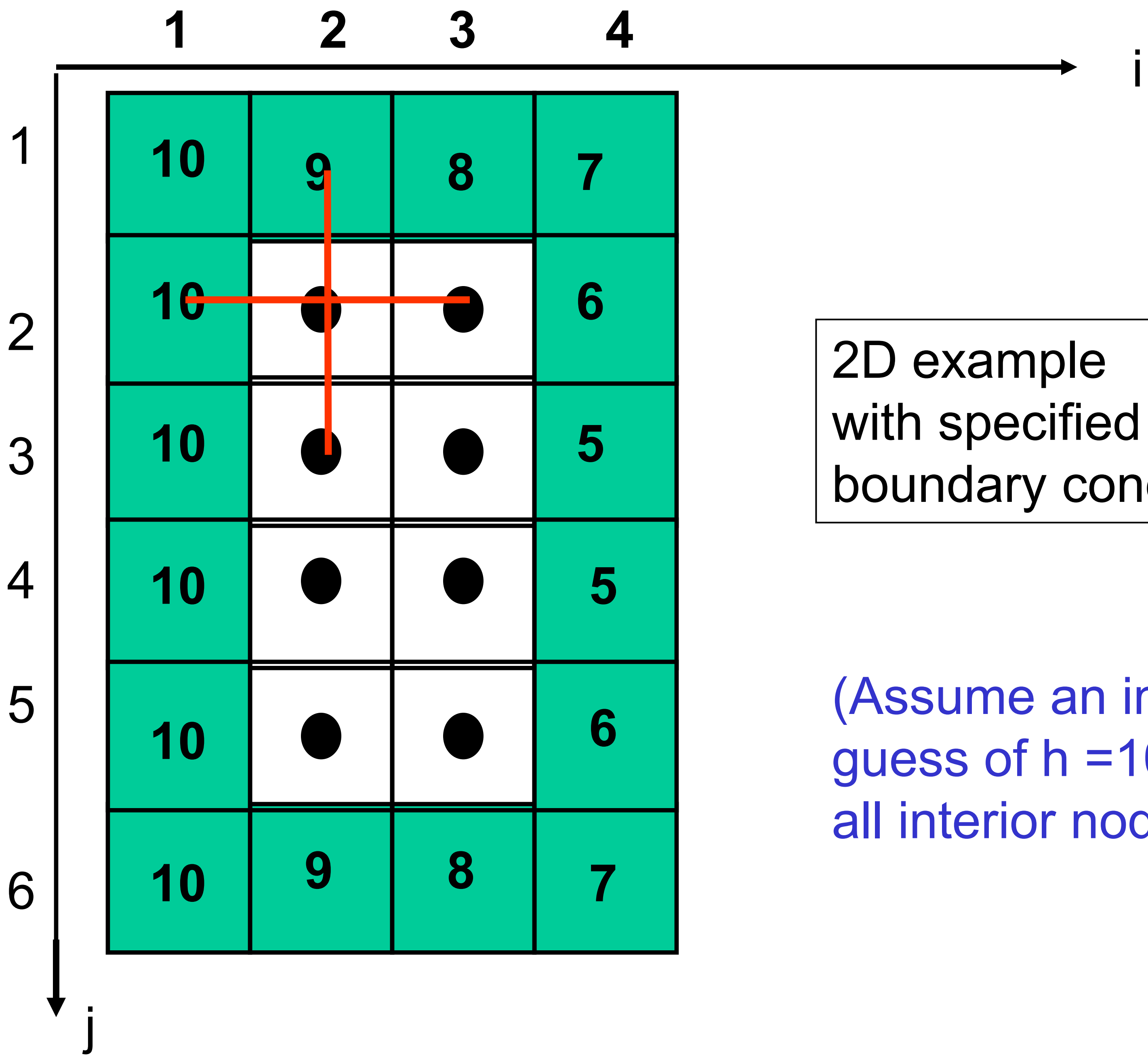


Iteration planes

$$h_{i,j}^{m+1} = \frac{h_{i+1,j}^{m} + h_{i-1,j}^{m} + h_{i,j+1}^{m} + h_{i,j-1}^{m}}{4}$$

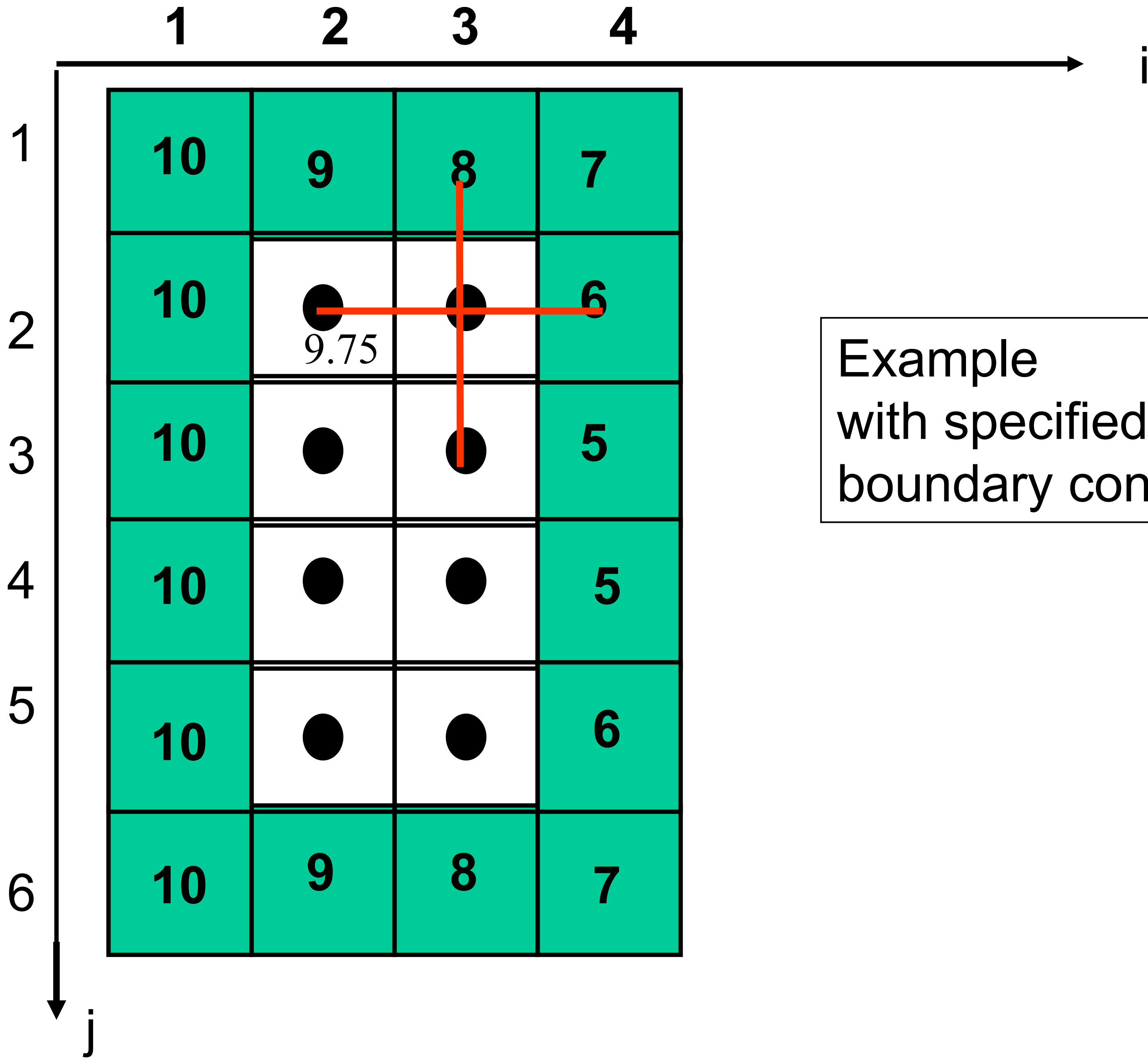
$$h_{i,j}^{m+1} = \frac{h_{i+1,j}^{m} + h_{i-1,j}^{m+1} + h_{i,j+1}^{m} + h_{i,j-1}^{m+1}}{4}$$

Gauss-Seidel Iteration

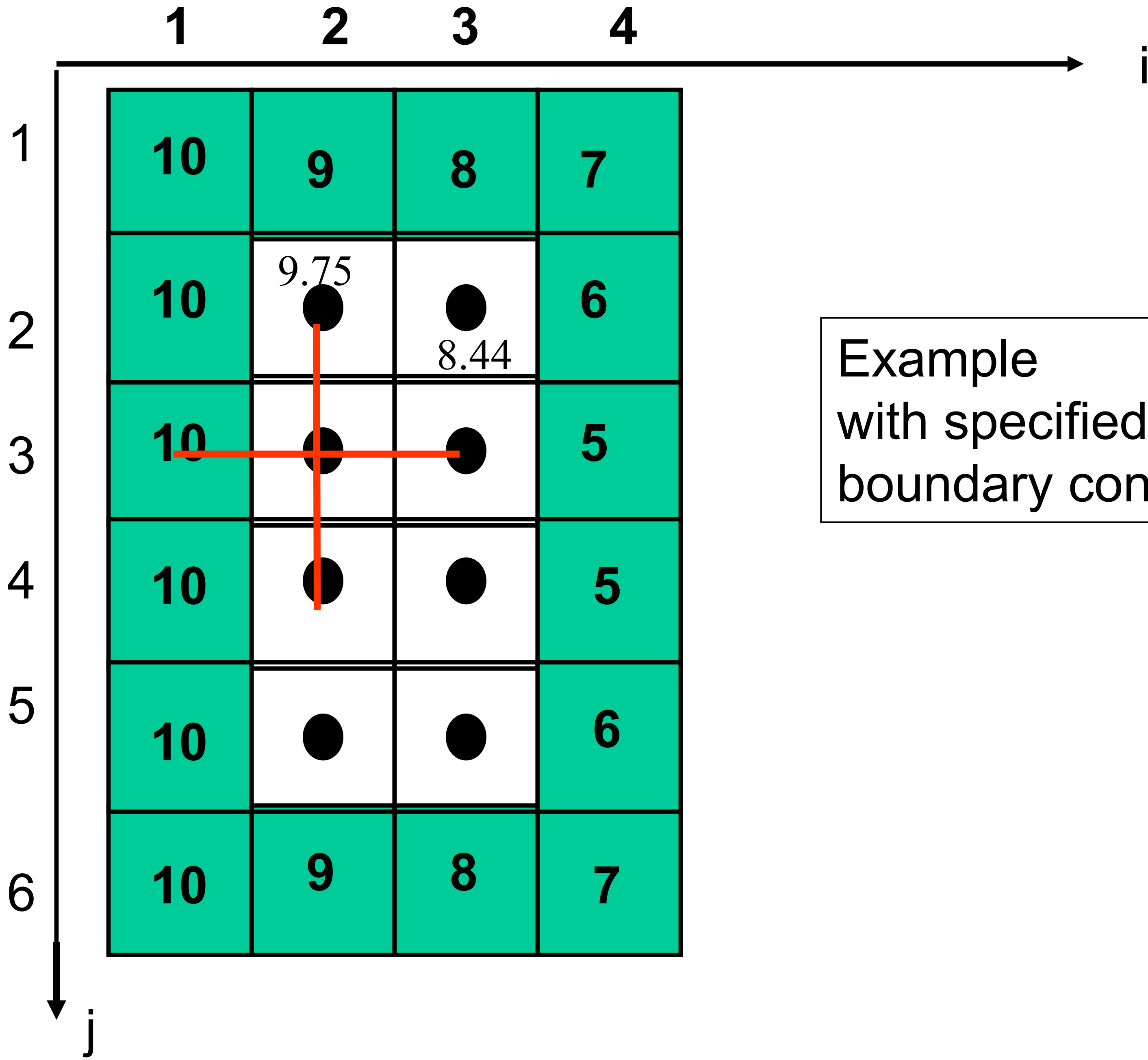


2D example
with specified head
boundary conditions

(Assume an initial guess of h =10 for all interior nodes)

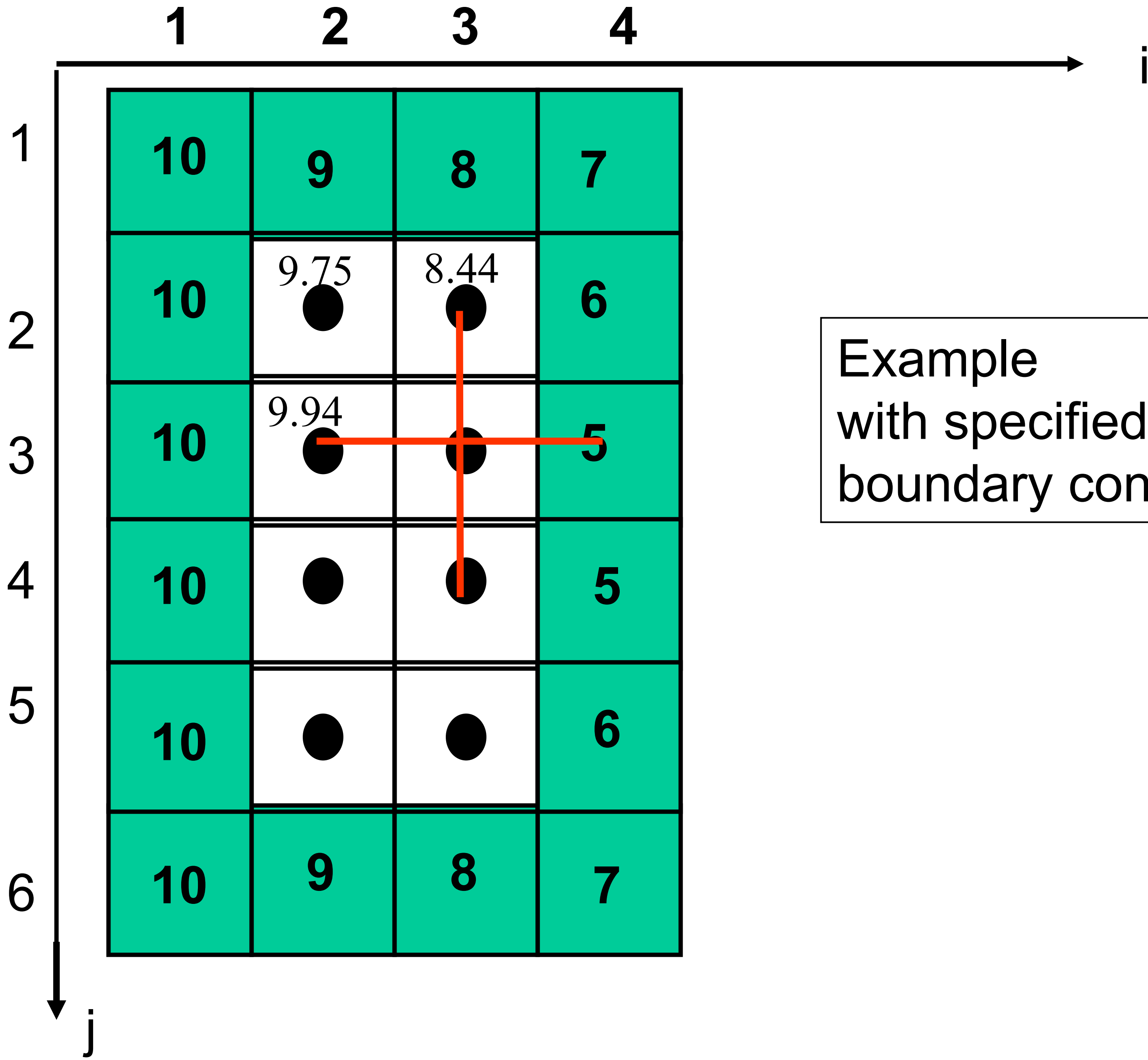


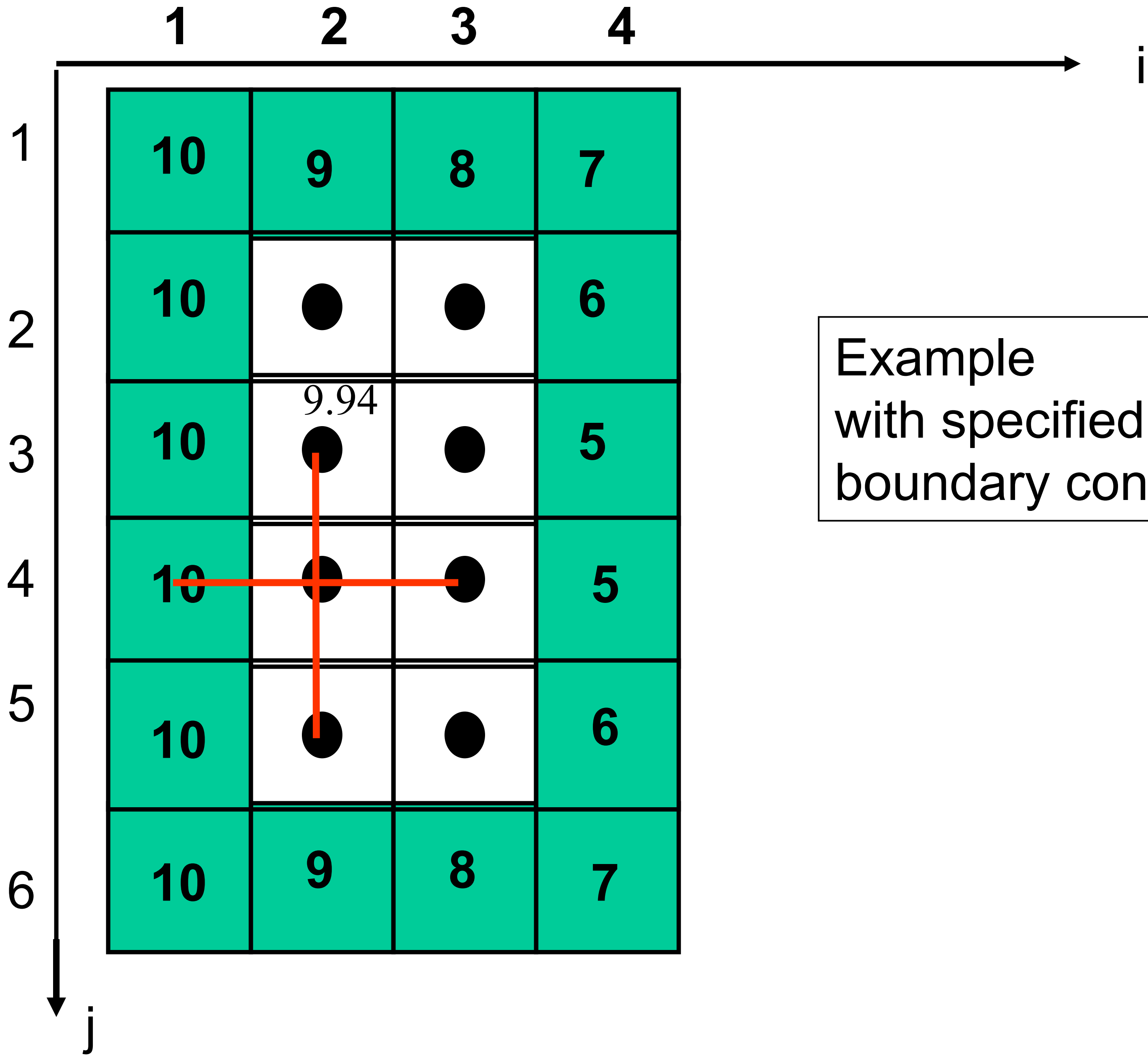
Example
with specified head
boundary conditions



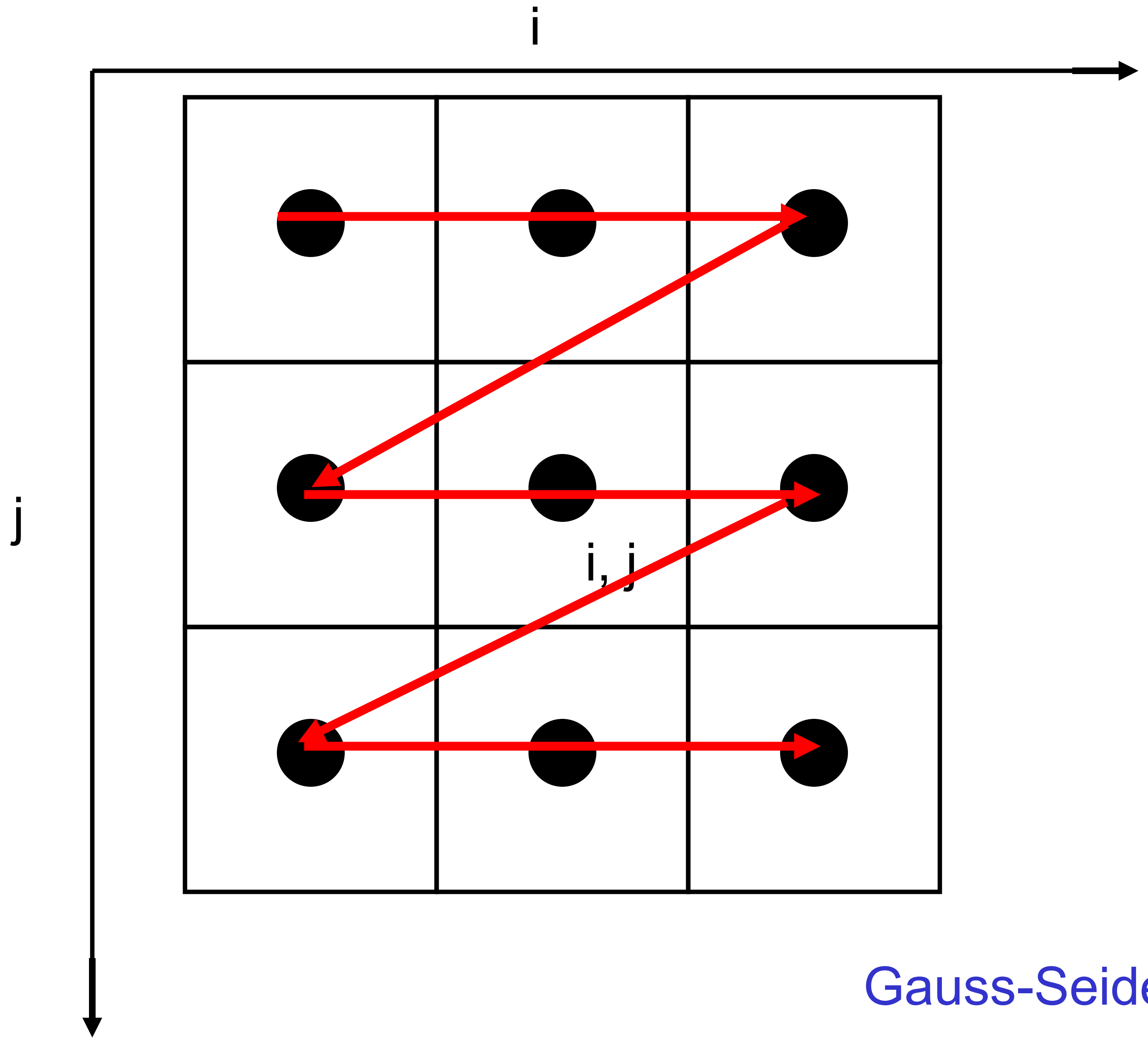
Example
with specified head
boundary conditions

Example
with specified head
boundary conditions

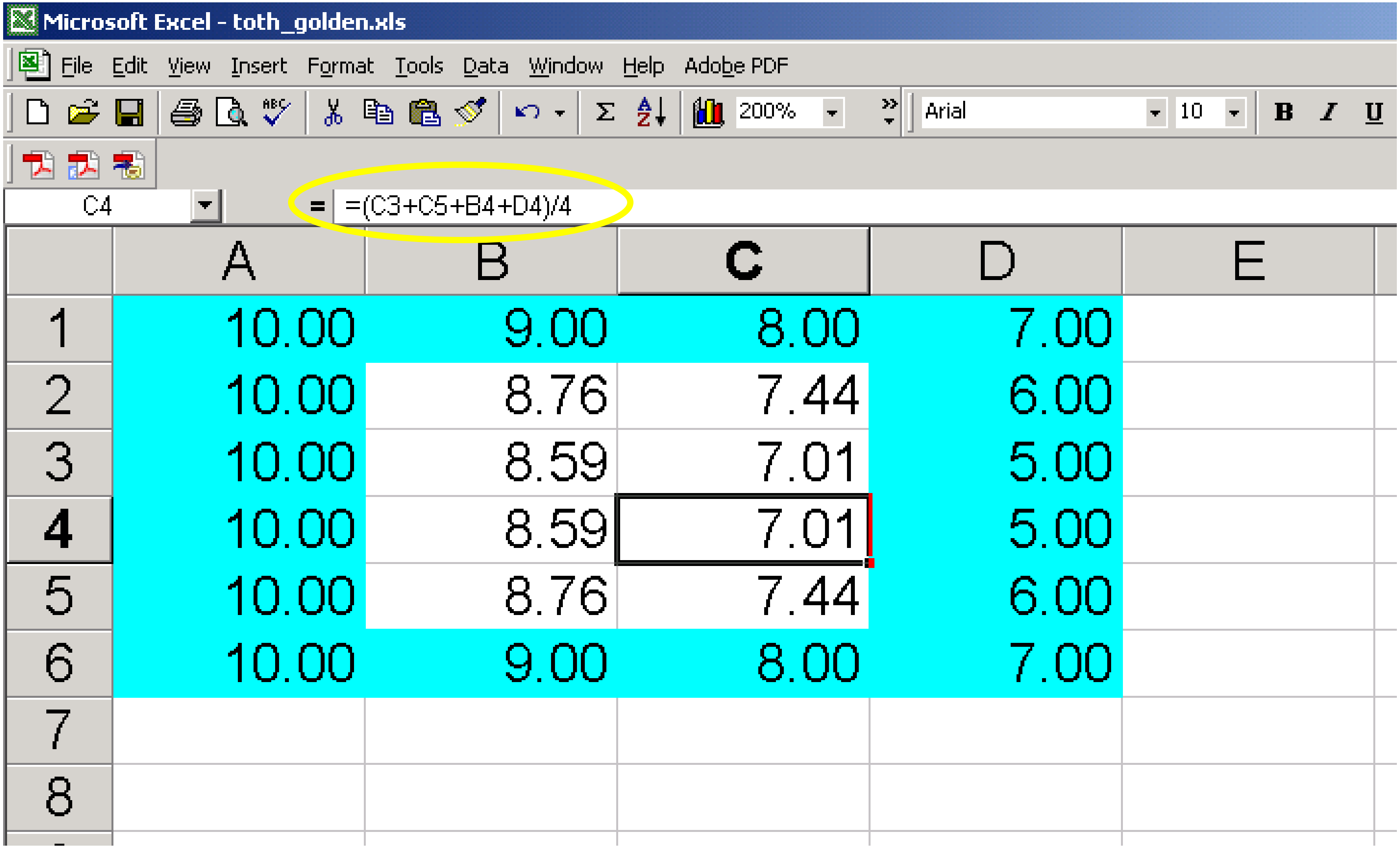




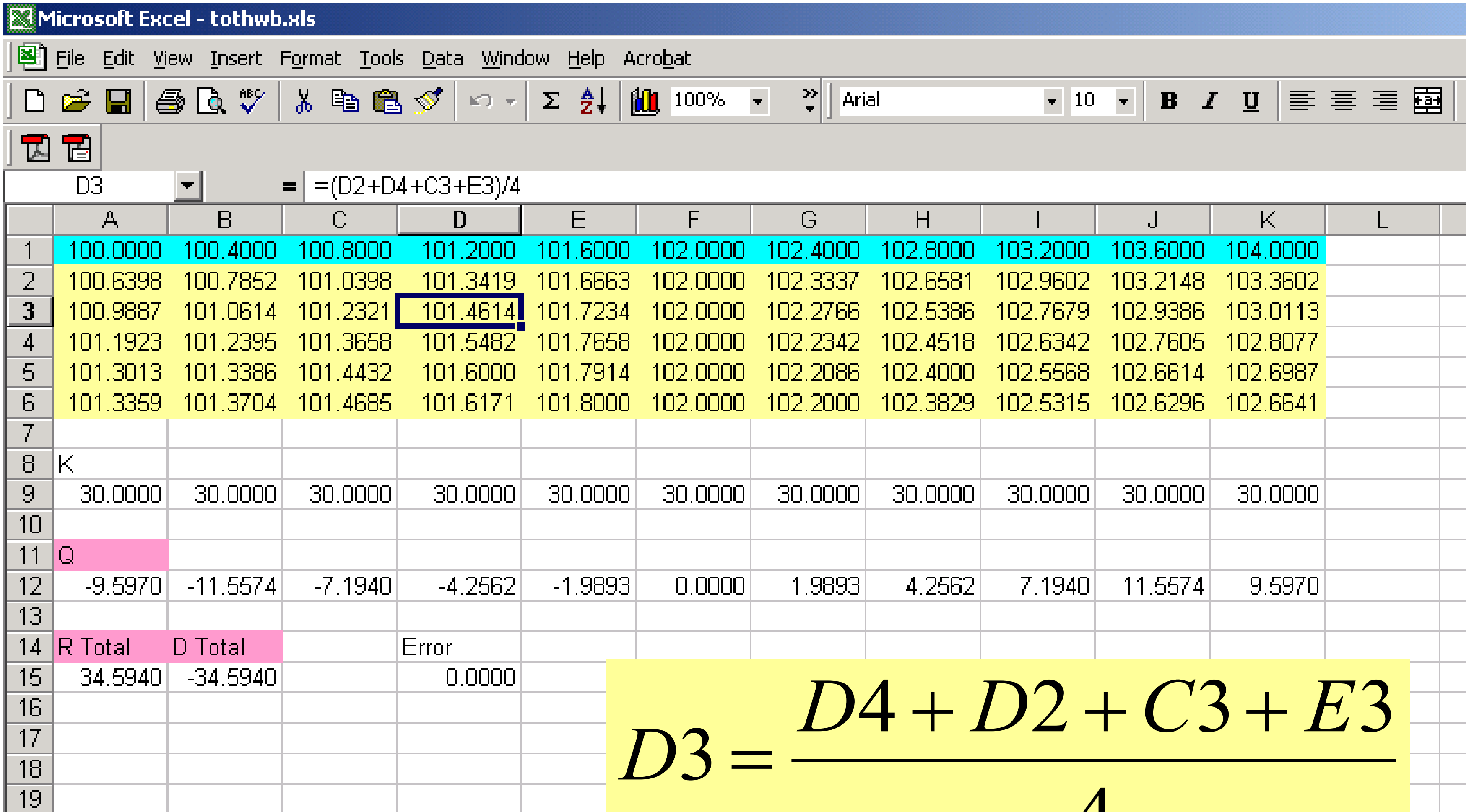
Example
with specified head
boundary conditions



Gauss-Seidel Iteration



C4=(C3+C5+B4+D4)/4



	A	B	C	D	E	F	G	H	I	J	K	L
1	100.0000	100.4000	100.8000	101.2000	101.6000	102.0000	102.4000	102.8000	103.2000	103.6000	104.0000	
2	100.6398	100.7852	101.0398	101.3419	101.6663	102.0000	102.3337	102.6581	102.9602	103.2148	103.3602	
3	100.9887	101.0614	101.2321	101.4614	101.7234	102.0000	102.2766	102.5386	102.7679	102.9386	103.0113	
4	101.1923	101.2395	101.3658	101.5482	101.7658	102.0000	102.2342	102.4518	102.6342	102.7605	102.8077	
5	101.3013	101.3386	101.4432	101.6000	101.7914	102.0000	102.2086	102.4000	102.5568	102.6614	102.6987	
6	101.3359	101.3704	101.4685	101.6171	101.8000	102.0000	102.2000	102.3829	102.5315	102.6296	102.6641	
7												
8	K											
9	30.0000	30.0000	30.0000	30.0000	30.0000	30.0000	30.0000	30.0000	30.0000	30.0000	30.0000	
10												
11	Q											
12	-9.5970	-11.5574	-7.1940	-4.2562	-1.9893	0.0000	1.9893	4.2562	7.1940	11.5574	9.5970	
13												
14	R Total	D Total		Error								
15	34.5940	-34.5940		0.0000								
16												
17												
18												
19												

$$D3 = \frac{D4 + D2 + C3 + E3}{4}$$

Example of spreadsheet formula

Toth Problem