Department of Electrical and Electronics Engineering

Overview

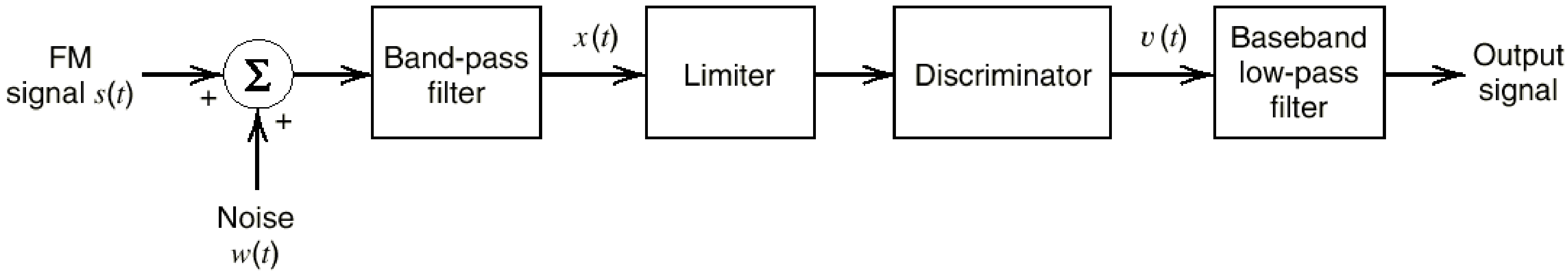
- Cochannel interference
- Analysis of Noise
  - Math model (good thing, not required)
  - Noise shape, preemphasis/deemphasis
  - FM threshold effects
- Security basics
- Homework 1 hints
- Good news: This is the last class for exam 1

Co-channel Interference

- Source: $A\cos w_c t$, Interference: $I\cos(w_c+w)t$
- $r(t)= A\cos w_c t+I\cos(w_c+w)t=E(t)\cos(w_c t+\psi)$
- $\psi=\tan^{-1}(I\sin wt/(A+I\sin wt))\sim=(I/A)\sin wt$
- PM: $y=(I/A)\sin wt$, FM $=(Iw/A)\cos wt$
- When A is large, suppress weak interference better than AM.
- Capture effect
  - Winner takes all
  - 35dB for AM
  - 6 dB for FM/PM
- White Gaussian noise
  - Noise increases linearly with frequency in FM.

System Model and Noise Model

Discriminator consists of a slope network and an envelope detector.



Let $n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$

$$n(t) = r(t)\cos\left[(2\pi f_c t) + \psi(t)\right]$$

The envelope is $r(t) = \left[n_I^2(t) + n_Q^2(t)\right]^{1/2}$

The phase is $\psi(t) = \tan^{-1}\left[\dfrac{n_Q(t)}{n_I(t)}\right]$

$$f_R(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right),\ r \ge 0$$

$$f_\Psi(\psi) = \frac{1}{2\pi},\quad 0 < \psi < 2\pi$$

where $r(t)$ is Rayleigh distributed, and $\Psi(t)$ is uniform distributed over 2π.

Signal after bandpass filter

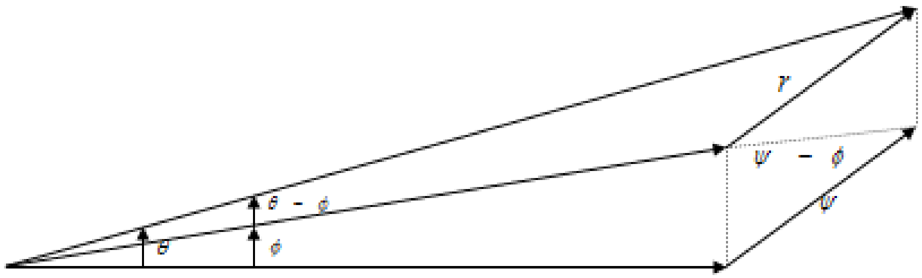
- The incoming FM signal $s(t)$ is defined by

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right]$$
$$= A_c \cos[2\pi f_c t + \phi(t)]$$

where $\phi(t) = 2\pi k_f \int_0^t m(\tau)\,d\tau$

- At the bandpass filter output

$$x(t) = s(t) + n(t)$$
$$= A_c \cos[2\pi f_c t + \phi(t)] + r(t)\cos[2\pi f_c t + \psi(t)]$$



where $A_c >> |r(t)|$

$$\theta(t) = \phi(t) + \tan^{-1}\left\{ \frac{r(t)\sin[\psi(t) - \phi(t)]}{A_c + r(t)\cos[\psi(t) - \phi(t)]} \right\}$$

Discriminator Output

- Note that the envelope of $x(t)$ is of no interest to us (limiter)

Because $A_c >> |r(t)|$

$$\theta(t) \approx \phi(t) + \frac{r(t)}{A_c} \sin[\psi(t) - \phi(t)]$$

$$= 2\pi k_f \int_0^t m(\tau) d\tau + \frac{r(t)}{A_c} \sin[\psi(t) - \phi(t)]$$

The discriminator output is

$$v(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}$$

$$= k_f \underbrace{m(t)}_{\text{message}} + \underbrace{n_d(t)}_{\text{additive noise}}$$

where

$$n_d(t) = \frac{1}{2\pi A_c} \frac{d}{dt} \{ r(t) \sin[\psi(t) - \phi(t)] \}$$

Noise After Discriminator

Assume $\psi(t)-\phi(t)$ is uniformly distributed over $(0, 2\pi)$,

then $n_d(t)$ is independent of message signal.

We may simplify $n_d(t)$ as

$$n_d(t) \approx \frac{1}{2\pi A_c}\frac{d}{dt}\{r(t)\sin[\psi(t)]\}$$

From definition of $r(t)$ and $\psi(t)$, we have

$$n_Q(t) = r(t)\sin[\psi(t)]$$

$$n_d(t) \approx \frac{1}{2\pi A_c}\frac{dn_Q(t)}{dt}$$

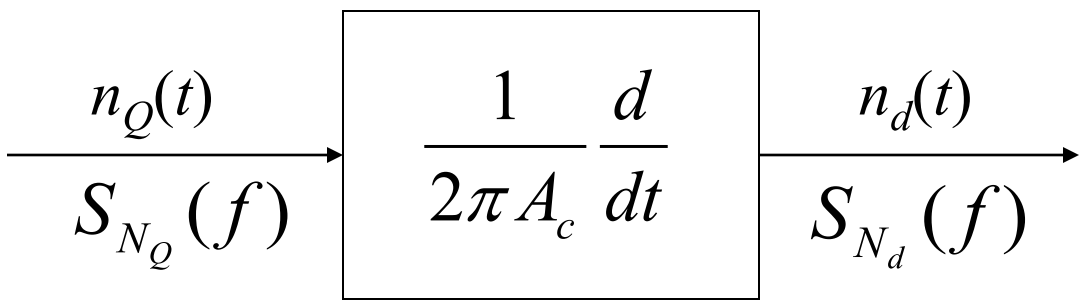
The quadrature appears

Noise After Discriminator cont.

- The average output signal power = $k_f^2 P$

Recall

$$\frac{d}{dt} \overset{F.T}{\Leftrightarrow} j2\pi f$$



$$S_{N_d}(f) = \frac{f^2}{A_c^2} S_{N_Q}(f)$$

Noise After Discriminator cont.

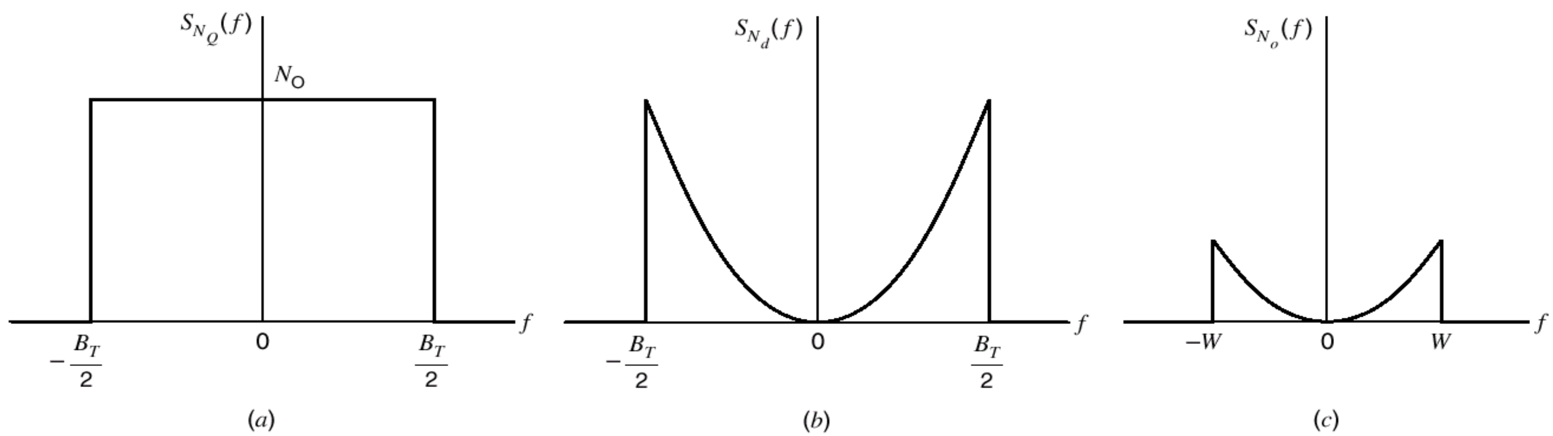
- Assume that $n_Q(t)$ has ideal low-pass characteristic with bandwidth B_T

$$S_{N_d}(f) = \frac{N_0 f^2}{A_c^2} \ , \quad |f| \le \frac{B_T}{2}$$

If $B_T/2 > W$

At thereceiver output

$$S_{N_0}(f) = \frac{N_0 f^2}{A_c^2}, \quad |f| \le W$$



SNR of FM

$$\text{Average power of } n_0(t) = \frac{N_0}{A_c^2}\int_{-W}^{W} f^2\,df$$

$$= \frac{2N_0W^3}{3A_c^2}$$

$$\propto \frac{1}{A_c^{\,2}} \text{ noise quieting effect}$$

$$(\text{SNR})_{O,\text{FM}} = \frac{3A_c^2k_f^2P}{2N_0W^3}$$

The average power of $s(t)$ is $\frac{A_c^2}{2}$,

the average noise power in message bandwidth is WN_0

$$\Rightarrow (\text{SNR})_{C,\text{ FM}} = \frac{A_c^2}{2WN_0}$$

$$\Rightarrow \left.\frac{(\text{SNR})_O}{(\text{SNR})_C}\right|_{\text{FM}} = \frac{3k_f^2P}{W^2}$$

Single Tone FM SNR

$$s(t) = A_c \cos\left[2\pi f_c t + \frac{\Delta f}{f_m}\sin(2\pi f_m t)\right]$$

We may write, $2\pi k_f \int_0^t m(\tau)d\tau = \frac{\Delta f}{f_m}\sin(2\pi f_m t)$

$\frac{d}{dt}$ both side $\Rightarrow \quad m(t) = \frac{\Delta f}{k_f}\cos(2\pi f_m t)$

The average power of $m(t)$ (across 1Ω load) is $P = \frac{(\Delta f)^2}{2k_f^2}$

From (2.149), $(\mathrm{SNR})_{O,\,\mathrm{FM}} = \frac{3A_c^2(\Delta f)^2}{4N_0W^3} = \frac{3A_c^2\beta^2}{4N_0W}$

$$\Rightarrow \left.\frac{(\mathrm{SNR})_O}{(\mathrm{SNR})_C}\right|_{\mathrm{FM}} = \frac{3}{2}\left(\frac{\Delta f}{W}\right)^2 = \frac{3}{2}\beta^2$$

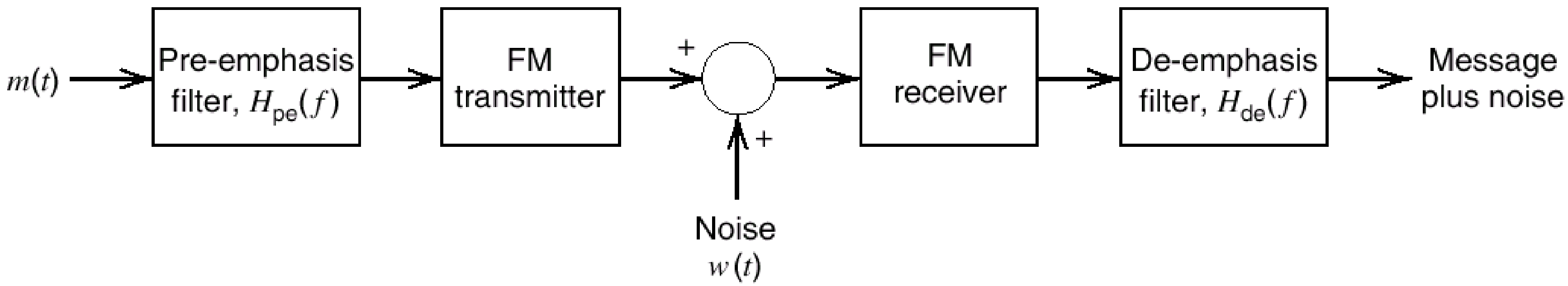
compare to AM, $\left.\frac{(\mathrm{SNR})_O}{(\mathrm{SNR})_C}\right|_{AM} = \frac{1}{3}$

When $\frac{3}{2}\beta^2 > \frac{1}{3}$, FM has better performance.

$\Rightarrow \beta > \frac{\sqrt{2}}{3} = 0.471$

Define $\beta = 0.5$ as the transition between narrowband FM and wideband FM.

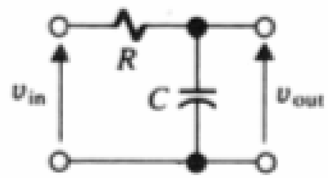
FM Preemphsis and Deemphasis



FM Preemphsis and Deemphasis

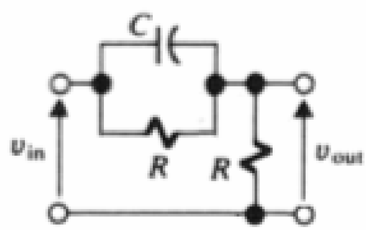
- FM related noise emphases can be suppressed by *pre-distortion* and post detection filters (preemphases and deemphases filters):

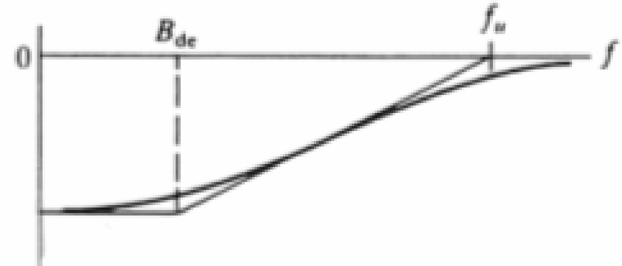
receiver filter



$H_{de}(f)$

transmitter filter





$H_{pe}(f)$

Q: What would happen if the filters would be reversed? (TX filter in receiver & vice versa)

$$H_{de}(f) = [1 + j(f / B_{de})]^{-1} \approx \begin{cases} 1, |f| << B_{de} \\ B_{de} / (jf), |f| >> B_{de} \end{cases}$$ LPF

$$H_{pe}(f) = [1 + j(f / B_{de})] \approx \begin{cases} j(f / B_{de}), f_u > |f| > B_{de} \\ 1, |f| >> f_u \end{cases}$$ HPF

THANKING YOU